USN ☐☐☐☐☐☐☐☐☐☐   **21MAT11**

First Semester B.E. Degree Examination, Jan./Feb. 2023
Calculus and Differential Equation

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Derive with usual notations $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$. **(06 Marks)**

   b. Prove that the curves $r^n = a^n \cos n\theta$ and $r^n = a^n \sin n\theta$ intersect orthogonally. **(07 Marks)**

   c. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at $(1, 1)$. **(07 Marks)**

OR

2 a. Derive an expression for radius of curvature in Cartesian form. **(06 Marks)**

   b. Find the angle of intersection between two curves $r = a \sec^2(\theta/2)$ and $r = b \csc^2(\theta/2)$. **(07 Marks)**

   c. Find the radius of curvature of the curve $r^2 = a^2 \cos 2\theta$. **(07 Marks)**

Module-2

3 a. Expand $\log(1 + \sin x)$ by Maclaurin's series upto 4th degree terms. **(06 Marks)**

   b. If $Z = xy^2 + x^2y$, where $x = at^2$, $y = 2at$, find the total derivative $\frac{dz}{dt}$. **(07 Marks)**

   c. Find the maximum value of the function $f(x, y) = x^3 y^2 (1 - x - y)$ for $x \ne 0$, $y \ne 0$. **(07 Marks)**

OR

4 a. Expand $\log(1 + e^x)$ using Maclaurin's series upto 4th degree terms. **(06 Marks)**

   b. Evaluate $\underset{x \to 0}{\text{Lim}}^{x \to 0} \left(\frac{\sin x}{x}\right)^{1/x^2}$. **(07 Marks)**

   c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$ and $w = x + y + z$, find the value of Jacobian $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. **(07 Marks)**

Module-3

5 a. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. **(06 Marks)**

   b. Find the orthogonal trajectories of the family of curves $x^2 + y^2 + 2\lambda x + C = 0$, λ - parameter. **(07 Marks)**

   c. Solve $x^2 p^2 + 3xyp + 2y^2 = 0$ where $P = \frac{dy}{dx}$. **(07 Marks)**

OR

6   a.   Solve $y(y + x)dx + (x + 2y - 1) dy = 0$.   (06 Marks)
    b.   A body originally at 80°C cools down to 60°C in 20 min, the temperature of the air being 40°C. What will be the temperature of the body after 40 min from the original?   (07 Marks)
    c.   Solve $(y - px)(p - 1) = p$ by reducing to Clairaut's form.   (07 Marks)

Module-4

7   a.   Solve $(D^3 - 4D^2 + 4D) y = 0$.   (06 Marks)
    b.   Solve $(D - 2)^2 y = 8(e^{2x} + 3)$.   (07 Marks)
    c.   Solve $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 3y = x^2$.   (07 Marks)

OR

8   a.   Solve $(D^3 - 6D^2 + 11D - 6)y = e^{2x} + e^{-3x}$.   (06 Marks)
    b.   Apply the method of variation of parameters to solve $\dfrac{d^2y}{dx^2} + 4y = 4\sec 2x$.   (07 Marks)
    c.   Solve $(1 + x)^2 \dfrac{d^2y}{dx^2} + (1 + x)\dfrac{dy}{dx} + y = 2\sin\log(1 + x)$.   (07 Marks)

Module-5

9   a.   Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$.   (06 Marks)

    b.   Apply Gauss-Jordan method to solve the system of linear equations:
         $x + y + z = 9$
         $x - 2y + 3z = 8$
         $2x + y - z = 3$ (07 Marks)
    c.   Use Gauss-Seidel method to solve the system of linear equations iteratively (3 iterations).
         $20x + y - 2z = 17$
         $3x + 20y - z = -18$
         $2x - 3y + 20z = 25$ (07 Marks)

OR

10 a.   Test the following system for consistency and solve if the system is consistent:
         $x + 2y + 3z = 1$
         $2x + 3y + 8z = 2$
         $x + y + z = 3$ (06 Marks)
    b.   Use Gauss elimination method to solve the system of equations
         $x + 4y - z = -5$, $x + y - 6z = -12$, $3x - y - z = 4$.   (07 Marks)
    c.   Determine the largest Eigen value and the corresponding Eigen vector of the matrix
         $$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$
         Choose initial eigen vector as $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^1$. Carryout 5 iterations.   (07 Marks)

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